

The motion and shape of a bubble in highly viscous liquid flowing through an orifice

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Highlights

- Observations show the shapes of a bubble, initially spherical, as it is taken through an orifice by a **very** viscous liquid.
- On approaching the orifice, the bubble elongates in the direction of the flow. Downstream of the orifice, the bubble contracts in the direction of the flow, giving ‘crescent-moon’ shapes.
- These deformations are well-predicted by theory, which assumes that bubble deformations are the same as for a liquid droplet replacing the bubble.

Abstract

Experiments and theory concern the behaviour of a small bubble carried through an orifice by a **very** viscous liquid. The liquid was polybutene oil, of viscosity about 70 Pa s, *i.e.* 70,000 times that of water. The Reynolds number of the flow is substantially less than one, hence the flow pattern is approximately radial flowing into, and away from, the orifice. These flow patterns have profound effects on the shape of an entrained bubble. On the upstream side, the acceleration of the liquid, as it approaches the orifice, causes elongation of the bubble since the front of the bubble moves faster than the back. On the downstream side, the reverse occurs: the back of the bubble moves fast than the front. Thus the height of the bubble diminishes as it moves away from the orifice, leading to the formation of a ‘crescent-moon’ shape. The shape of these bubbles can be predicted by considering the motion of a droplet of the same liquid replacing the bubble: the resulting geometric theory gives good predictions of bubble deformation approaching the orifice and of ‘crescent-moon’ formation downstream of the orifice.

1. Introduction

Highly viscous liquids containing gas bubbles are used to make foams. It therefore seems appropriate to study the behaviour of a single bubble in a very viscous liquid, flowing through an orifice, to answer questions as follows. How does the bubble deform on passing through the orifice? Does the bubble change shape? Does it break up and, if so, how?

These questions were answered by using a multipass rheometer (MPR) (Mackley et al., 1995): the MPR contains two pistons and cylinders, the pistons moving to-and-fro synchronously, so as to induce to-and-fro motion of liquid through an orifice connected to the two cylinders. The orifice was constructed of stainless steel, but held in place by quartz glass windows such that bubbles in the liquid could be observed as they deformed. The liquid used was a polybutene viscosity standard (Brookfield): a Newtonian fluid with a viscosity of 68 Pa s. This liquid was chosen to explore bubble deformation in capillary number regimes that cannot be obtained with lower viscosity media, such as water, and is representative of the viscosity of some polymer foams at high shear rate (Wang, 2009).

The motion and behaviour of single bubbles and clusters of bubbles was observed by high speed photography. Remarkable changes of bubble shape were observed when a single bubble passed through an orifice:

- i. as a bubble approached the orifice, the bubble elongated in the direction of flow;
- ii. after a bubble had passed through the orifice, the liquid motion on the downstream side caused the bubble to take on the shape of a crescent moon;
- iii. rather than undergoing break-up, the crescent-shaped bubble returned to its initially spherical shape significantly downstream of the orifice.

A simple kinematic theory is presented in this paper that is able to give insight into these experimental observations: it assumes that the bubble deformation is the same as that of a ‘marked drop’ of liquid embedded within the parent liquid. The key assumption underlying this theory is that the bubbles are effectively ‘passive markers’ embedded within the liquid phase, corresponding to a capillary number regime where viscous effects are dominant.

2. Literature survey

Fluids consisting of gas-liquid mixtures are widespread in both nature and industry. Bubbly liquids, typically containing around 50% by volume of liquid (Torres et al., 2015), are encountered in volcanic magmas (Mader et al., 2013), in foamed cements (Amran et al., 2015) and in the food industry (Mezzenga et al., 2005). Foams are classified either as ‘wet’, with liquid content ranging between 15% and 36% on a volumetric basis, or dry, where the liquid content is typically less than 5% (Drenckhan and Hutzler, 2015). Uses of foam are widespread in engineering (Eaves, 2004), the food industry

(Narsimhan and Xiang, 2018) and are commonly encountered in the natural world (Hill and Eastoe, 2017).

Often, bubbly liquids or wet foams are precursors to the formation of porous solids. The gas bubbles become voids within a matrix of solidified liquid and serve to enhance the properties of the resultant material. For example: foamed concrete has a lower density than its non-voided counterpart (Amran et al., 2015), making it more suitable for cementing oil wells in fragile rock structures, and foamed polyurethane has a thermal conductivity an order of magnitude lower than that of the single phase polymer (Collishaw and Evans, 1994). The ultimate properties of the porous solid are dependent on those of the single-phase materials and on the structure that results from the distribution of voids within the solid.

The behaviour of gas bubbles contained within a flowing liquid is, therefore, of relevance to the design of ‘engineered’ porous solids. This has been the subject of research spanning many decades: phenomena such as Ostwald ripening (Ostwald, 1897; Voorhees, 1985), bubble breakup and bubble coalescence (Prince and Blanch, 1990) all affect the structure and distribution of the gas phase. Moreover, when bubbles rise through a liquid due to buoyancy there are fundamental differences between the dynamics of a single bubble (Wallis, 1974) and swarms of bubbles (Richardson and Zaki, 1954). It is within the context of this background that we wish to address the questions posed in the introduction.

As the viscosity of the liquid phase increases, and as the effect of inertia diminishes, bubble shape is governed primarily by the ratio of viscous deformation forces to surface tension forces, described by the capillary number, Ca . Of particular relevance to the work presented here are theoretical and experimental studies concerning the deformation of bubbles as they flow through ducts and constrictions at low Reynolds number and with ‘moderate’ to ‘high’ capillary number.

Theoretical studies of the deformation of bubbles in drops in various flow fields have been well-summarised in the literature (Rallison, 1984). Studies involving simple shear flows are of relevance: slender body theory has been developed to describe the deformation of droplets as $Ca \rightarrow \infty$ (Hinch and Acrivos, 1980), with Rallison providing theory to describe droplet deformation at arbitrary capillary number (Rallison, 1984). This latter study concluded that low viscosity drops (*i.e.* bubbles) can attain highly stable, elongated, shapes: these shapes may have pointed tips, or cusps. Extensive theoretical investigation of the formation and stability of cusped interfaces has subsequently been carried out (Pozrikis, 1998; Shikhmurzaev, 1998; Siegel, 2000).

Long, slender bubbles having cusped tips have been observed experimentally in simple shear flows over the range $0.01 < Ca < 10$ (Rust and Manga, 2002): these observations were in good agreement with theory. Further research by the same authors identified the presence of similarly-shaped bubbles in a solidified magma (Rust et al., 2003). Aerofoil-shaped bubbles, with just a single cusp, have been

observed in pipe flow for $Ca > 1$ (Kameda et al., 2008). The shape of these bubbles departed from the twin-cusp slender bodies observed in simple shear flow due to the fully three-dimensional Poiseuille nature of the pipe flow: theory developed for two-dimensional Poiseuille flow (Richardson, 1973) was able to capture some of the observed features. Observations of bubbles flowing through a Hele-Shaw cell, where $6 \times 10^{-5} < Ca < 2 \times 10^{-2}$, revealed a rich variety of shapes such as elongated bubbles, ‘short-tail’ and ‘long-tail’ bubbles (Kopf-Sill and Homsy, 1988): the broad spectrum of bubble shapes was attributed to the fully three-dimensional nature of the flow field.

The flow-field produced as liquid flows through a constriction or an orifice is substantially more complex than that for a simple shear flow or a Poiseuille flow: the flow-field will contain extensional and shear components and the capillary number will vary as a function of location. Measurements of deformation and velocity have been made for a bubble contained with a flow of 1.05 Pa s silicone oil as it passes through a narrow orifice (Dawson et al., 2015). Bubble elongation upstream of the orifice was observed, with bubbles taking mildly concave shapes downstream of the orifice. Under some circumstances, cusp formation was observed as the bubble passed through the orifice.

A theoretical investigation of the behaviour of liquid droplets suspended in a continuous liquid phase flowing through a rounded constriction has been done by Khayat and co-workers (Khayat et al., 2000). For the case where the viscosity ratio between the continuous and discrete phases is 4, they compute the formation of highly deformed ‘crescent-moon’ droplets if an initially spherical droplet is placed in the centre of the constriction prior to the commencement of flow. Zero surface tension was assumed, in this study hence $Ca = \infty$. Shapes similar to those observed by Dawson were also computed for an initially spherical droplet placed upstream of the constriction prior to the commencement of flow. Experimental work has recently been undertaken (Sauzade and Cubaud, 2018) that examines the deformation of bubbles as they pass through a series of rounded constrictions, similar to those used in the theoretical study of Khayat and co-workers. Crescent-moon shaped deformation was not observed in this geometry, probably due to the nature of the periodically constricting channel, but a number of differently-shaped bubbles containing cusps were observed.

3. Experimental

3.1 Apparatus

Figure 1(A) shows the essential features of the apparatus. The viscous liquid is shown in white. The orifice plate is shown in black within the black, dashed, square. Two pistons, indicated in black, go back and forth synchronously, pushing the liquid to-and-fro through the orifice plate.

<FIGURE 1>

The apparatus is quasi ‘two-dimensional’: the orifice and the region marked in white contained in the plane of the diagram are sandwiched between two glass windows. These windows, separated by a

distance of 1 mm normal to the plane of the diagram, set the thickness of the flow channel. The pistons, black in Figure 1(A) are cylindrical, of diameter 10 mm, with a connecting piece to transfer the liquid from the cylinders to the observation region. The width of the channel immediately before and after the orifice is the same as the piston diameter, *i.e.* 10 mm. The shape of the flow constriction results from two stainless steel inserts, which are sandwiched between the glass windows. These inserts are interchangeable and can take the form of an orifice, shown, or contraction-expansion channels with various contraction angles. The orifice plate that is illustrated in Figure 1(B) has an opening 1 mm wide and plate thickness of 1mm.

Pressures were measured by a pair of pressure transducers (Omega XXXXXX) installed at both ends of the flow chamber. Each pressure transducer contained an integrated thermocouple; there was also a separate thermocouple at the centre of the flow chamber. The light source and camera, shown in Figure 1(A), were positioned to observe the flow constriction such that the axis joining the light source and camera was normal to the plane of the diagram.

A high speed CMOS camera (Basler acA1300-200HC) was used with an LED light source (Thorlabs MCWWHCP1) to obtain the bubble images. The test liquid was a Newtonian polybutene oil (Brookfield viscosity standard B7300, 73.0 Pa s at 25 °C). A rotational rheometer (TA Instruments ARES) was used to verify the viscosity of the test liquid: the results, which are available in the supplementary material¹, demonstrated that the liquid was Newtonian with an apparent viscosity of 67 Pa s. The difference from the 73.0 Pa s quoted above is probably due to temperature effects.

3.2 *Experimental procedure*

Polybutene oil was first loaded into the test chamber of the MPR, shown in white in Figure 1(A). Air bubbles were then injected into the chamber using a syringe with a hypodermic needle. These bubbles were stably suspended in the liquid: the effects of buoyancy and of Ostwald ripening were slow due to the liquid's high viscosity. The flow chamber was formed by selecting different quartz glass windows and stainless steel inserts to obtain the desired geometry of the flow passage: this could take the form of the orifice indicated in Figure 1(B) or a convergent-divergent nozzle.

The upper and lower pistons can be controlled, either individually or together, to generate the desired pressure and flow rate within the observation chamber. Piston velocities ranged from 0.01 to 10 mm/s, corresponding to volumetric flow rates ranging between 0.785 to 785 mm³/s. Images of the bubble motion were recorded using the high speed camera and the pressure difference across the flow chamber was measured.

¹ Note to editor: supplementary material will be provided if this paper is accepted for publication.

The bubble dynamics and shape deformation were obtained from the bubble images and subsequent image analysis. The open source image analysis software FIJI (Schindelin et al., 2012) and Icy (de Chaumont et al., 2011) were used: data extracted from these tools was further processed using Origin® (OriginLab).

4. Theory

4.1 Geometric theory for bubble deformation

Figure 2 shows the basis of the theory. Figure 2(A) illustrates the flow pattern schematically: the flow field is assumed to be two-dimensional and radial, converging to and diverging away from the orifice marked 'S'. At a given radius, r , from S the radial velocity, u_r , is assumed to be constant. From continuity, the product of the radial velocity and the radial distance, $u_r r$, is constant.

<FIGURE 2>

The second assumption concerns the behaviour of the bubble, shown in Figure 2(B) and Figure 2 (C). It is assumed that the bubble boundary, initially circular when viewed in cross section, moves as in the same manner as a region of the continuous phase marked with dye. This assumes that the effect of surface tension is small, corresponding to large capillary number, and that the effect of constant pressure within the bubble is negligible in comparison with the pressure variations within the continuous phase. This latter assumption is likely to be valid when the bubble diameter is small compared with the magnitude of the distances covered by the bubble when approaching and leaving the orifice.

The assumption that the bubble behaves in the same manner as an initially circular element of the continuous phase leads to a simple geometric theory to describe its shape at different points in the flow. A detailed derivation of this theory is given in the Appendix, but the principle is described here. Consider points A and B on the bubble surface, shown in Figure 2(A). The assumption of continuity, *i.e.* that the product $u_r r$ is constant leads to Equation A2 that relates the radius, r , to the time, t . This leads to Equation A4, which enables the relative movements of the front and back of the bubble to be predicted. Thus in Figure 2 (A), A moves to A' and B moves to B'; likewise P moves to P' and Q moves to Q'. The length A'B' must be less than the length AB because the back of the bubble always moves faster than the front, *i.e.* the velocity of A' is greater than that of B'.

This velocity difference leads to the formation of 'crescent moon' bubbles shown in Figure 2(C): the successive positions of A are shown by the asterisk; likewise the successive positions of B are shown by the plus sign. The same analysis shows that for a bubble drawn towards the orifice, *i.e.* the upstream side of S in Figure 2 (A), the bubble must elongate in the direction of flow. This follows from the fact that the front of the bubble moves faster than the back. The equations that describe the shapes shown in Figure 2 (C) are given in the Appendix.

4.2 Image analysis

The ‘Active Cells’ plug-in (Delgado-Gonzalo et al., 2015) was used with Icy (de Chaumont et al., 2011) to carry out image analysis using the fast active contour method (Williams and Shah, 1992). The aim of the analysis was to obtain a set of co-ordinates defining the interface of the bubble in each of the sequence of images that were acquired. These data were then further processed using Origin® to give three measures of shape that are commonly used for discrete particles, roundness, elongation and convexity ratio (Mora and Kwan, 2000): these three measures are illustrated schematically in Figure 3.

<FIGURE 3>

5. Results

5.1 Bubble shapes: experimental

Figure 4 is a sequence of photographs showing the appearance of an initially spherical bubble being driven through an orifice by an upward flow. Although the apparatus was quasi two-dimensional, the initial bubble was spherical and of diameter 745 μm : this is of the same order of size as the orifice (1.0 mm). However, the bubble shapes observed exhibit forms similar to those predicted by the theory in Section 4. As the bubble is dragged upwards by the flow, it elongates in the direction of the flow, images (A) to (D). On passing through the orifice, the crescent shape emerges, images (F) to (H).

<FIGURE 4>

The remarkable variety of bubble shapes illustrated in Figure 4 share features observed by other authors. The ‘egg-shaped’ bubble in Figure 4(B) is similar to that shown by Dawson and co-workers (Dawson et al., 2015); the shape in Figure 4(D) is reminiscent of the ‘long-tail’ bubbles observed in a Hele-Shaw cell (Kopf-Sill and Homsy, 1988) and the ‘crescent-moon’ shapes in Figure 4(F) and (G) are similar to those resulting from the calculations by Khayat and co-workers (Khayat et al., 2000). Single cusps are visible in Figure 4(D), (E) and (F), with twin cusps being visible in Figure 4(H).

Figure 5 shows a sequence of images for a bubble passing twice through the orifice. Sequence (A), with downward flow, shows an initially elongated bubble passing down through an orifice. As it enters the orifice, the elongation increases, as expected. Then the bubble emerging downwards, shows the crescent shape, as expected. Sequence (B) shows the crescent-shaped bubble transform to an elongated bubble during up flow, as indicated by the red arrow in (B). But on emerging from the orifice, in the last frame on the right, the bubble again becomes crescent shaped.

<FIGURE 5>

Again, a remarkable variety of bubble shapes are visible in Figure 5. The images in Figure 5(A) can be considered to derive from essentially the same experiment as illustrated in Figure 4, with single cusp

‘long-tail’ bubbles and twin cusp ‘crescent-moon’ bubbles being observed as before. The images in Figure 5(B) can be compared to those in observed in a channel containing multiple constrictions (Sauzade and Cubaud, 2018), notably the ‘wedge-shaped’ bubble in Figure 5(B-iii) and the ‘plug-shaped’ bubble in Figure 5(B-iv). The influence of the initial bubble shape on the subsequent bubble deformation and breakup behaviour is also abundantly clear in Figure 5. Not only is there significant differences between the bubble shapes in Figure 5(B) when compared to Figure 5(A), but some breakup of the bubble is observed as it exits the orifice in the upstream direction, Figure 5(B-v) and Figure 5(B-vi).

Descriptions of droplet breakup for viscosity ratios as low as 10^{-2} (Stone, 1994) and critical capillary numbers for droplet breakup at viscosity ratios as low as 10^{-3} (Bentley and Leal, 1986) are given in the literature: the viscosity ratio in this experiment is 7×10^{-6} . Grace gives details of the breakup of initially spherical droplets in both shear and extensional flows with viscosity ratios as low as 10^{-6} for shear flow and 10^{-4} for extensional flows: critical capillary numbers of approximately 300 and 2, respectively, are required for breakup (Grace, 1982). The flow in Figure 5 contains both shear and extensional components, hence the critical capillary numbers quoted are likely to be the upper and lower bounds. The question of why droplet breakup is observed in Figure 5(B) and not in Figure 5(A), when one could reasonably argue that the capillary numbers are essentially the same in both experiments, can be answered by examining the difference in initial bubble shape. Stegeman and co-workers (Stegeman et al., 2008) build on the work by Grace by investigating the effect of initial droplet shape: they conclude that droplets that are initially stretched will breakup at capillary numbers lower than their spherical counterparts for the same viscosity ratio.

5.2 Bubble shapes: theoretical

Figure 6 shows theoretical predications for the shape of a two-dimensional bubble passing through an orifice: three stages can be identified.

- (i) As the bubble moves up towards the orifice (red), it elongates, becoming taller and thinner.
- (ii) When the bubble is part way through the orifice (green) the bit below the orifice continues to elongate. The bit above the orifice widens out.
- (iii) When the bubble is wholly through the orifice (blue), the widening is dominant leading to the crescent-moon shape.

<FIGURE 6>

5.3 Bubble shapes: experiment and theory compared

The shapes observed in experiments, Figure 4 and Figure 5, are qualitatively similar to those in Figure 6, despite the differences in the geometry between experiment and theory. Recall that (i) the experimental orifice shown in Figure 4 and Figure 5 has a finite height : width ratio whereas the theory

treats the orifice as a combined point source and point sink and (ii) that the initial bubble in the experimental case, shown in Figure 4, is spherical whereas it would be cylindrical in the two-dimensional theory. Motivated by the qualitative similarities, quantitative analysis of the bubble shape, both experimental and theoretical, was carried out.

<FIGURE 7>

Figure 7 shows the three measures of bubble shape, roundness, elongation and convexity ratio, for an initially spherical bubble passing upwards through the orifice. The blue lines represent the experiment illustrated in Figure 4 and the red dashed lines predictions of bubble shape from the geometric theory. Also marked on this diagram are the approximate times of each of the images shown in Figure 4. The following observations can be made:

- i. no single measure of bubble shape is sufficient to unambiguously describe the shape of the bubble;
- ii. elongation ratio yields similar numerical values both upstream and downstream of the orifice despite strikingly different bubble shapes;
- iii. a combination of roundness and convexity can be used to reasonably infer the shape changes the bubble undergoes: bubble stretching upstream of the orifice results in small values of roundness, but unity convexity ratio; within the orifice, the stretched bubbles return to a more spherical shape, shown by increasing roundness but still essentially unity convexity ratio; downstream of the orifice, as ‘crescent-moons’ form, both roundness and convexity ratio values decrease;
- iv. the roundness and convexity ratio values computed for the geometric theory qualitatively capture some of the trends shown experimentally, whereas elongation values remain high due to the smaller radii of curvature resulting from the geometric theory.

<FIGURE 8>

Figure 8 shows shape measurements for an initially crescent-moon shaped bubble flowing upwards through an orifice. In a similar manner to Figure 7, blue lines represent the experiment illustrated in Figure 5(B), with the approximate times of each of the frames marked, and red dashed lines are predictions from the geometric theory. Similar observations can be made to those resulting from Figure 7, notably (i) that a combination of roundness and convexity ratio appears to give good quantitative information concerning the change of bubble shape and (ii) that elongation ratio and roundness measurements derived from the geometric theory again qualitatively capture some of the trends observed experimentally.

5.4 Displacement of bubble boundaries and capillary numbers

Figure 9 shows the displacements, Y , of different parts of the bubble as it passes through the orifice. Note that on the upstream sides, the back of the bubble gets left behind the front due to elongation of the bubble. On the downstream side, the tips of the bubble get left behind as the crescent-shaped bubble gets wider.

<FIGURE 9>

Figure 10 shows capillary numbers, $Ca = \mu u / \gamma$, calculated as a function of time, for different parts of the same bubble. Velocity data was obtained from displacement data by use of finite differences. The capillary number indicates the relative importance of viscous and surface tension forces. In the upstream and downstream region surface tension dominates and hence $Ca < 1$. In the region of the orifice, however, $1 < Ca < 5$ so viscous forces are more important than surface tension. This may explain why the geometric theory shown in Figure 2 and Figure 6 works well: if surface tension is relatively unimportant, each bubble can deform readily to the crescent shapes.

6. Conclusions

A gas bubble in a liquid of large viscosity shows remarkable changes of shape as it is carried through an orifice by the liquid flow. With polybutene, whose viscosity is about 70,000 times that of water, the bubble elongates as it is moved towards the orifice. On emerging from the orifice, the bubble distorts into the shape of a crescent moon.

These changes of shape can be predicted by a simple kinematic model. The liquid flow converges as it approaches the orifice. The closer to the orifice, the higher the velocity thus a bubble approaching an orifice is elongated in the direction of the flow. After the bubble has passed through the orifice, it is in a region of diverging flow: the further from the orifice, the slower the velocity. Hence the back of the bubble is catching up with the front, giving the crescent moon shape, as observed.

These considerations led to a simple geometric theory to predict the bubble shape as it approaches and leaves the orifice. This theory gives reasonably good predictions of bubble shape. Differences between the theory and experiment are inevitable because:

- i. the theory assumes two-dimensional geometry, so the initial bubble shape is assumed to be cylindrical;
- ii. in the experiments, the liquid flowed between two parallel plates, so the velocity near the wall was small. The theory assumed two-dimensional flow, *i.e.* no velocity variation in the direction normal to the plane of observation.

Simple measures of bubble shape, namely roundness, elongation and convexity ratio, were calculated using data obtained from image analysis. Elongation alone gives ambiguous results but a combination

of roundness and convexity ratio were found to be appropriate measures for the bubble shapes that were encountered.

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8. Attributions

Data accessibility. All datasets presented in this work would be available on request.

Author's contributions. The project was conceived by JFD, CHC and BH. The experimental work was done by CHC. The theory was developed by JFD. The text was written by JFD and BH. The literature survey was done by BH. Diagrams were produced by JFD, CHC and BH.

Competing interests. The authors have no competing interests.

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9. Nomenclature

Roman letters

c	-	Integration constant for Equation (2)	(m ²)
k	-	Normalised volumetric flow per unit width, Q/π	(m ² /s)
Q	-	Volumetric flow rate per unit width	(m ² /s)
R_0	-	Initial radius of spherical bubble	(m)
r	-	Radius from orifice S	(m)
t	-	Time	(s)
u_r	-	Radial velocity	(m/s)
u	-	Velocity	(m/s)
Y	-	Bubble displacement	(m)

Greek letters

γ	-	Surface tension	(N/m)
μ	-	Viscosity	(Pa s)

Non-dimensional groups

Ca	-	Capillary number, $\mu u/\gamma$	(-)
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10. References

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11. Appendix

Consider the plate PP containing a small hole, S, that is shown in Figure 2(A). When highly viscous liquid is forced through S, we assume that the flow is radially inwards on the upstream side, and radially outwards on the downstream side. In the experiments described in this paper, the liquid viscosity was 70 Pa s. The forces arising on the bubble from viscosity will therefore be significantly larger than those resulting from surface tension.

The prediction of bubble shapes in this analysis assumes that the motion of the bubble periphery is the same as for a circular slug of dye put into the liquid. The subsequent motion of the dye in the liquid is assumed to be the same as that of the bubble surface. This assumption is likely to be true for very small bubbles with zero surface tension.

The equations derived below are for ‘two-dimensional’ motion, *i.e.* all velocities are in the plane of the diagrams. In Figure A 1, we follow a bubble, initially of radius R_0 , in radial flow away from a point source, S. Now consider the velocities. The volumetric flow rate for a unit thickness is Q m²/s, hence $Q = \pi r u_r$, r being the radius measured from S and u_r is the radial velocity at radius r . Thus

$$u_r = \frac{k}{r} = \frac{dr}{dt} \quad (1)$$

where $k = Q/\pi$. Integrating Equation (1) with respect to r gives

$$r^2 = 2kt + c \quad (2)$$

where c is a constant of integration. Equation (2) applies to all elements, so the relationship between initial and final positions are governed by this expression. Thus, for an element starting at position B,

$$r_3^2 - r_B^2 = 2kt(t_3 - t_B) \quad (3)$$

where r_3 is the final position of element B. The times t_B and t_3 refer to points B and 3. The time interval, $(t_3 - t_B)$ is the same for all elements, so

$$r_3^2 - r_B^2 = r_2^2 - r_F^2 = r_1^2 - r_0^2 \quad (4)$$

The term $r_1^2 - r_0^2$, relating the edge of the bubble, can be chosen to give a series of positions for the bubble so

$$\begin{aligned}
r_2^2 &= r_F^2 + r_1^2 - r_0^2 \\
r_3^2 &= r_B^2 + r_1^2 - r_0^2
\end{aligned}
\tag{5}$$

Equations (5) determines the shape of the bubble. For large values of r_0 , the term $r_1^2 - r_0^2$ will dominate, so r_3 will not be much less than r_2 , giving the crescent shapes shown in the photographs. The crescent shape arises from the fact that the back of the bubble moves faster than the front, because $u_r = k/r$ from Equation (1).

Following the analysis above, for a bubble passing through an orifice S, the bubble shape downstream can be predicted from the movements shown in Figure A 2. Point T moves to R, point B moves to r_3 , r_0 moves to r_1 , C moves to D and F moves to r_2 . From the fact that all elements have the same time lapse for the bubble, the above argument gives

$$\begin{aligned}
r_1^2 &= r_C^2 + r_D^2 - r_0^2 \\
r_2^2 &= r_C^2 + r_D^2 - r_F^2 \\
r_3^2 &= r_C^2 + r_D^2 - r_B^2 \\
r_R^2 &= r_C^2 + r_D^2 - r_T^2
\end{aligned}
\tag{6}$$

The procedure is to choose a succession of values of r_D ; for each r_D Equations (6) will give r_1 , r_2 , r_3 and r_R . As before, if r_D is large, r_3 will be only a little less than r_2 , giving a crescent-shaped bubble.

Consider Figure A 3. Here, the bubble is in converging flow, but no part of it reaches the orifice: thus the bottom of the bubble, point D, does not reach S. Note that the converging flow elongates the bubble, *i.e.* its height becomes greater than its width. Because of the converging flow, the bottom of the bubble travels faster than the top, hence explaining the elongating effect. This shows up in the 'pointed' form of the bottom of the bubble, as seen in the experiments and in Figure 6.

It is possible, though not too straightforward, to show that as a bubble distorts in converging or diverging flow, its area is constant. This is as it should be, for the above analysis simply follows the motion of liquid elements as if the gas bubble was not there. The above analysis can also be readily adapted to the case where the bubble is part way through the orifice; the top part of the bubble is in converging flow, as in Figure A 3. The lower part of the bubble is in diverging flow, as shown in Figure A 1, this resulting in a crescent-shaped form on the downstream side of the orifice.

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Figures

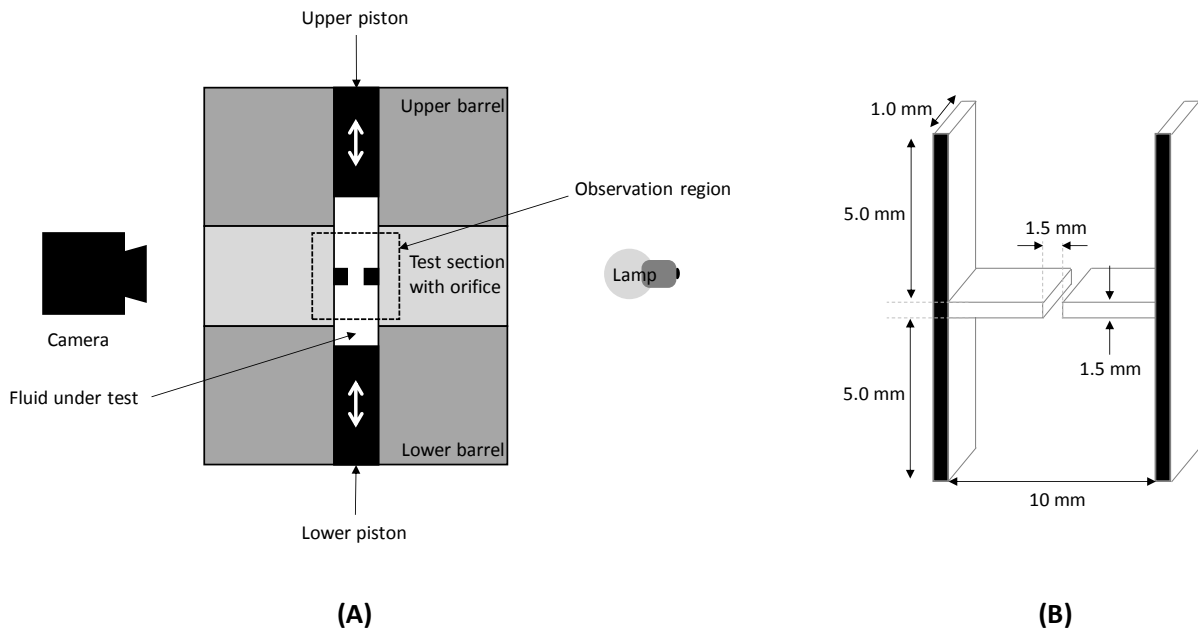


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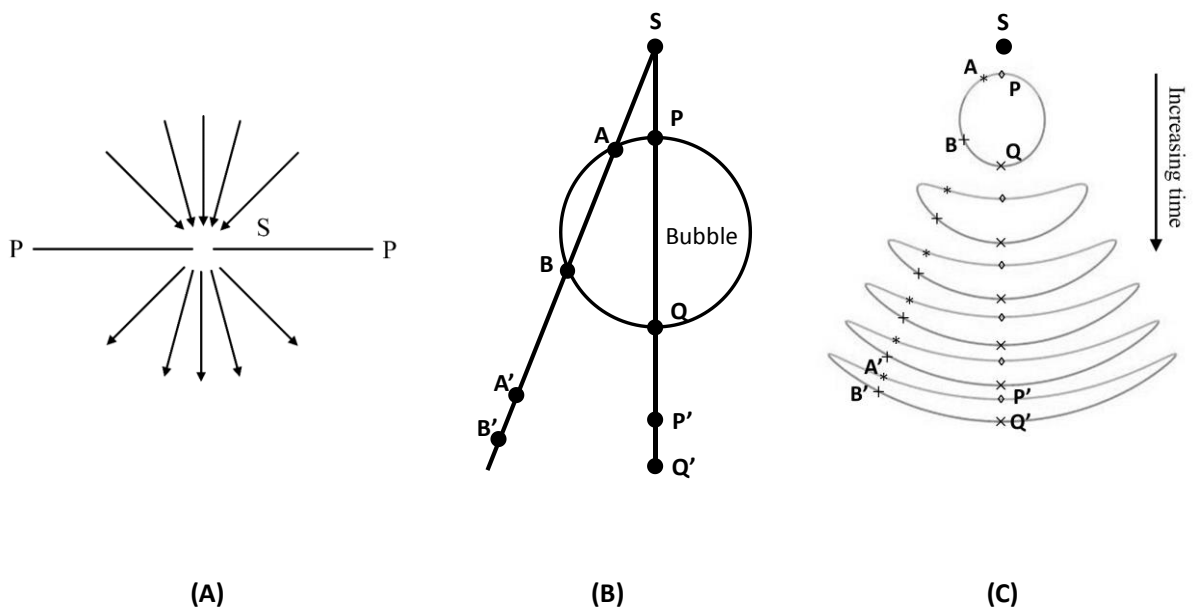


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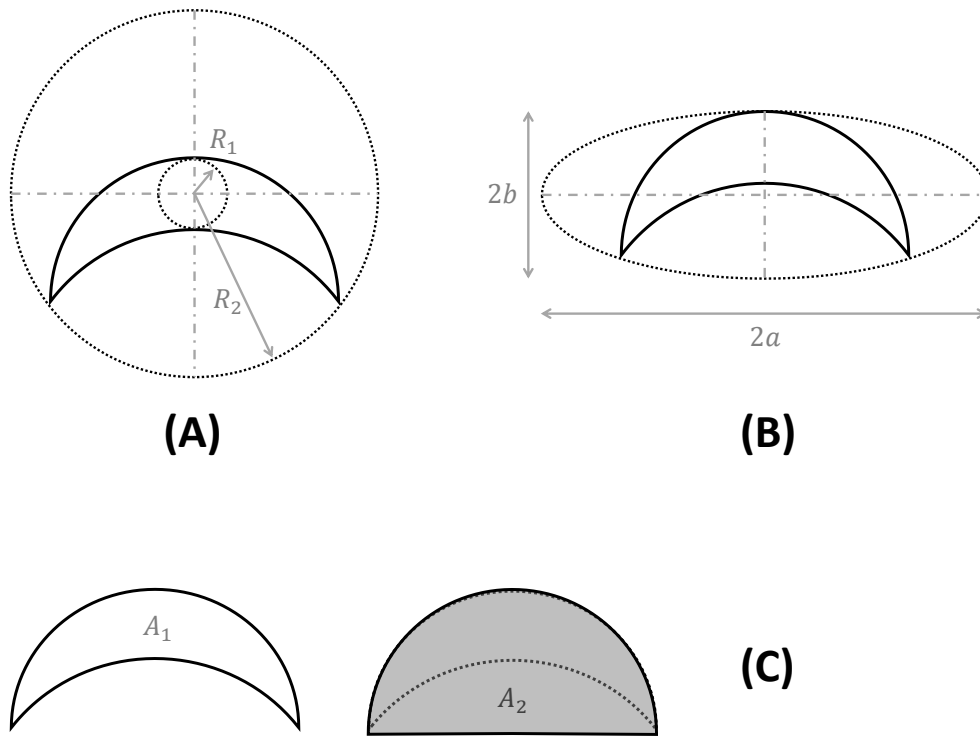


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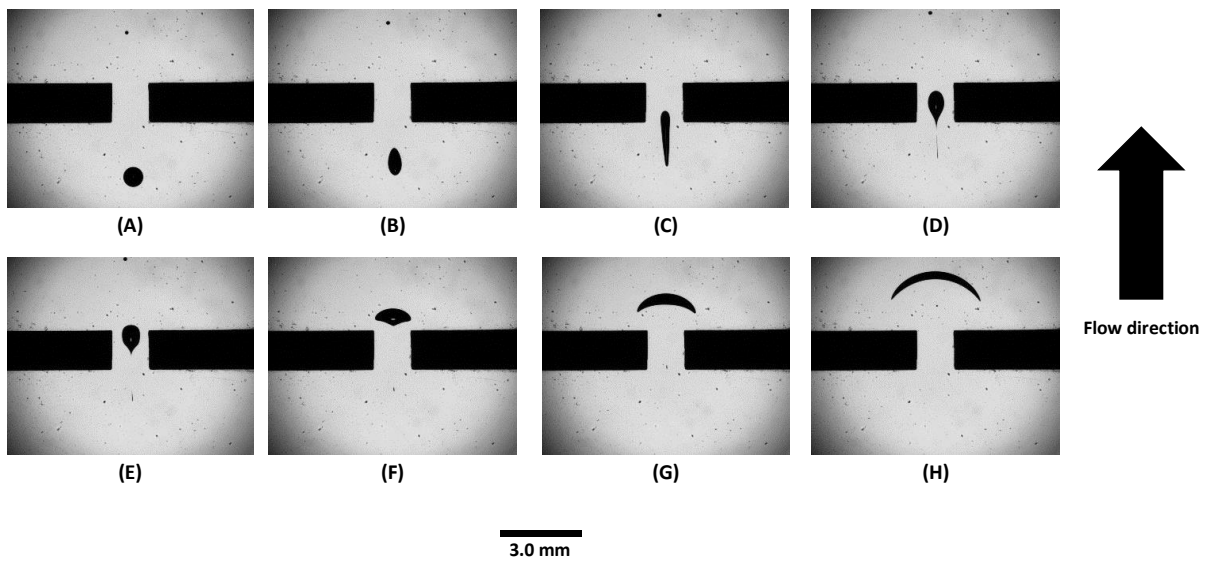


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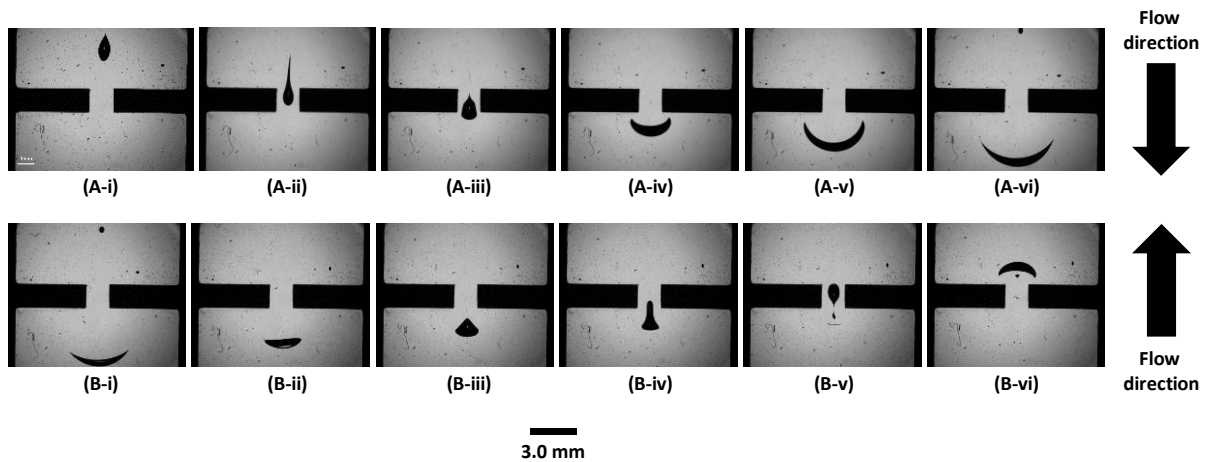


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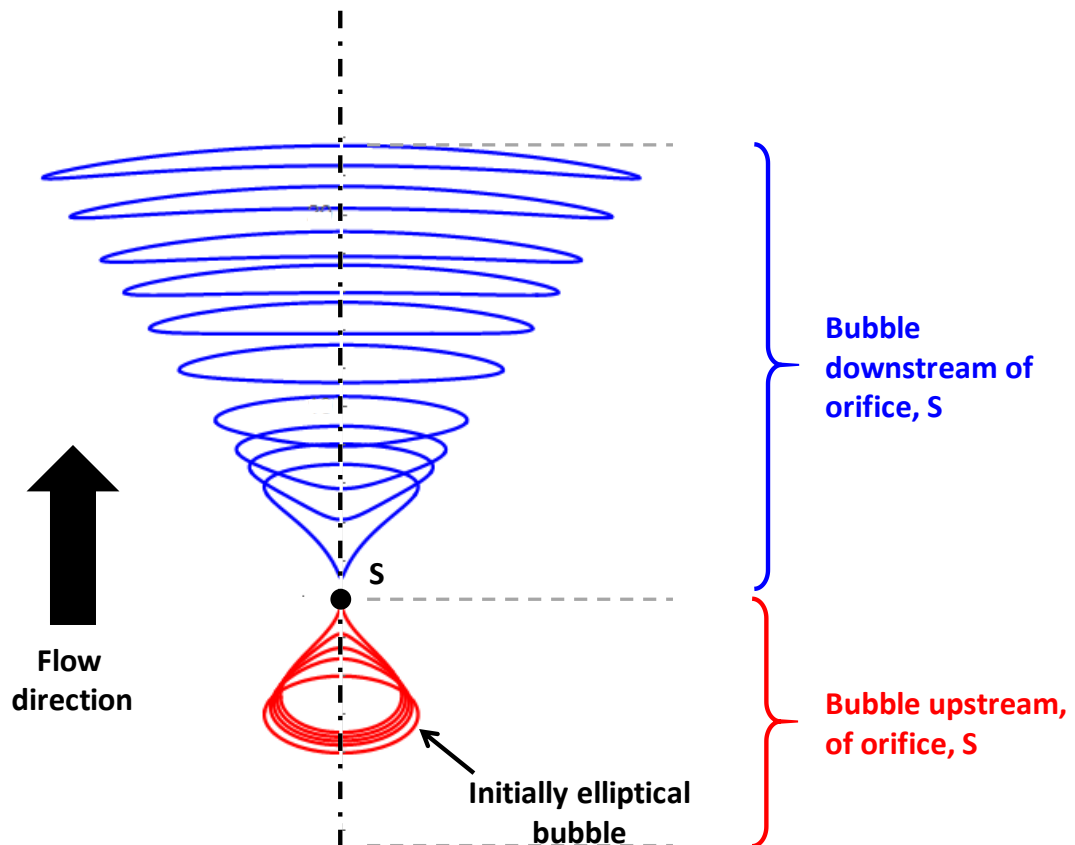


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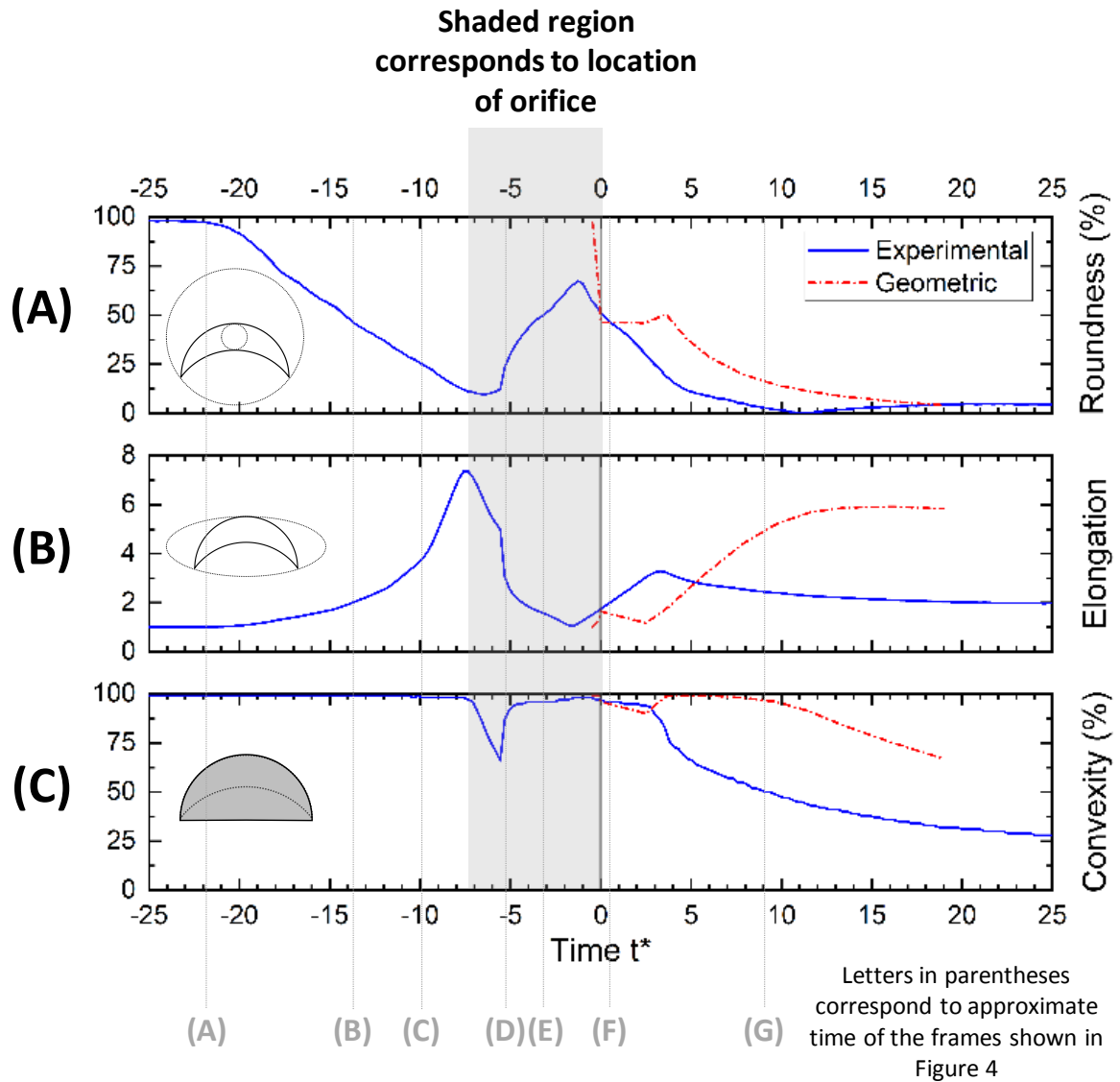


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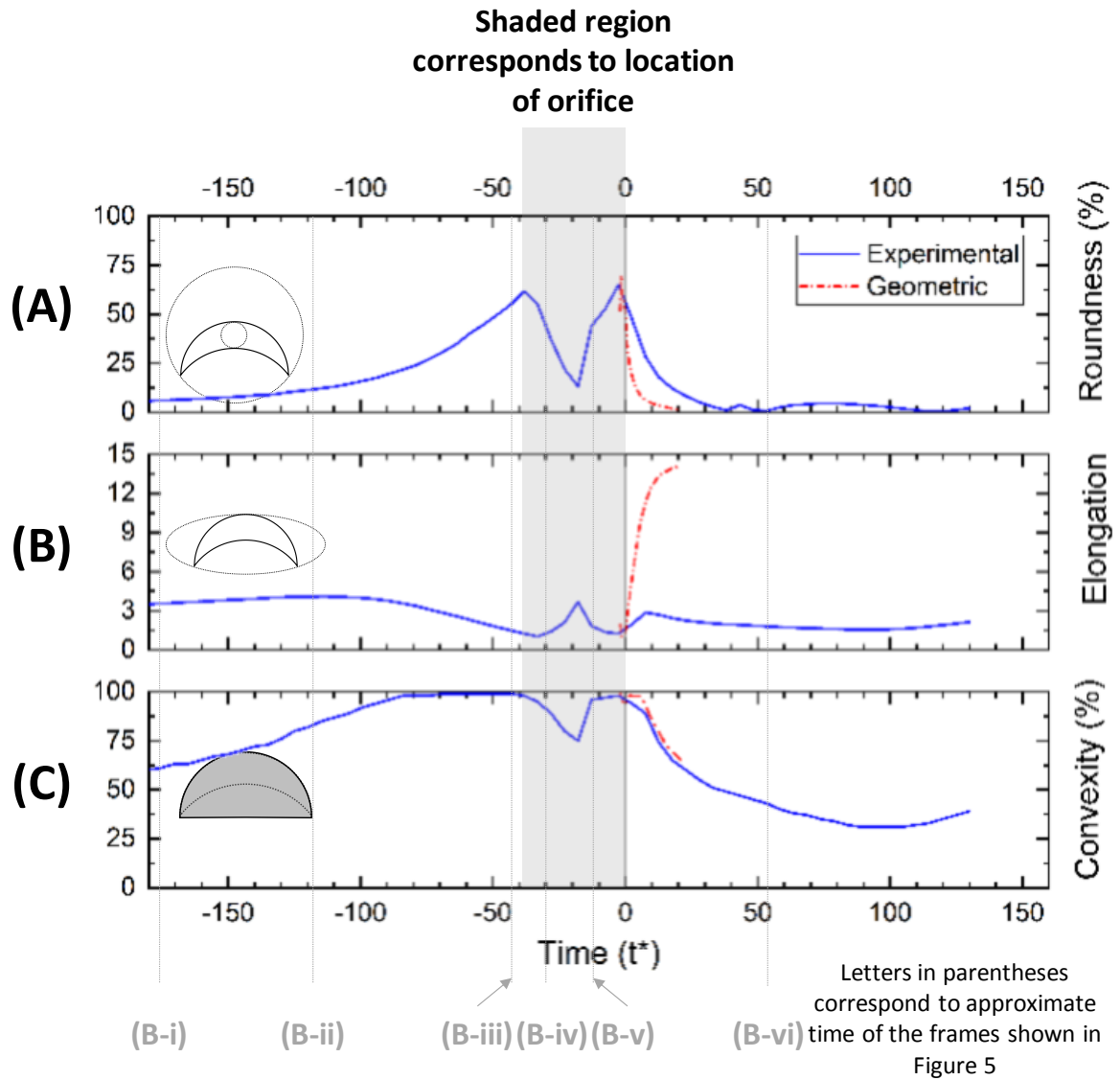


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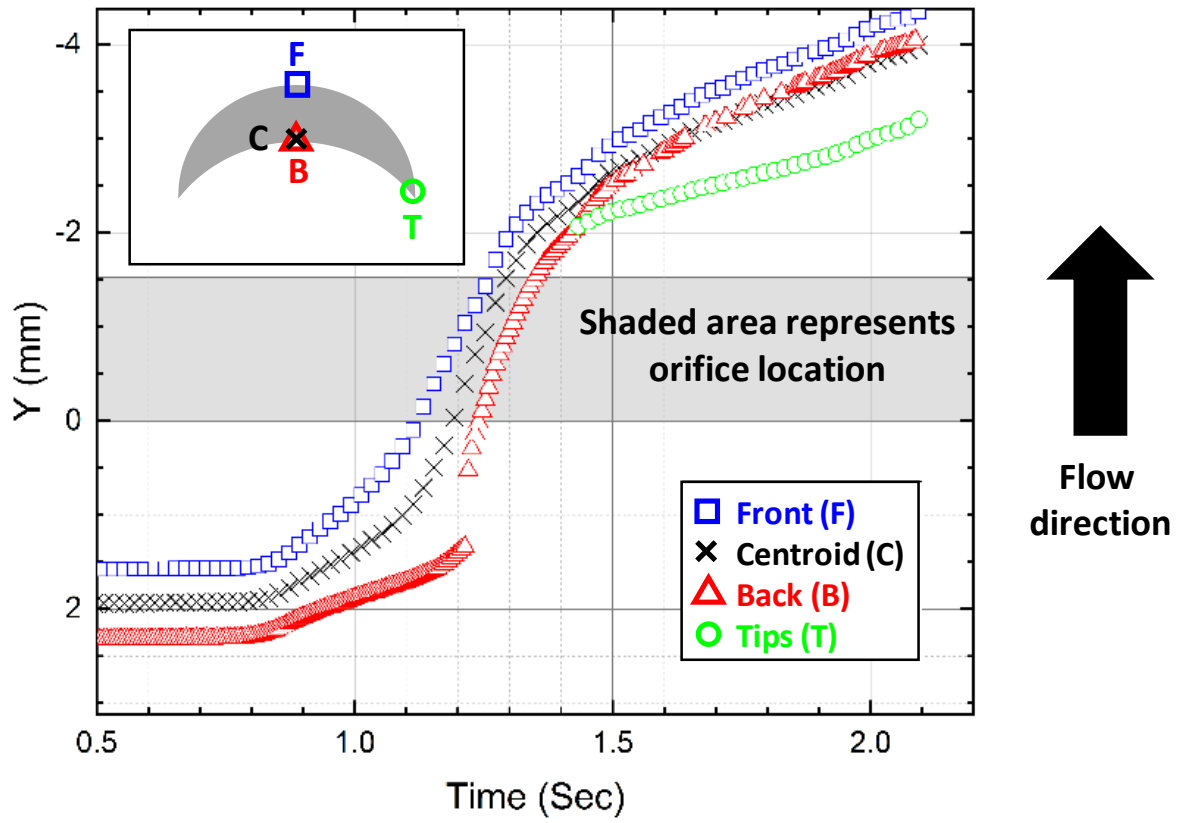


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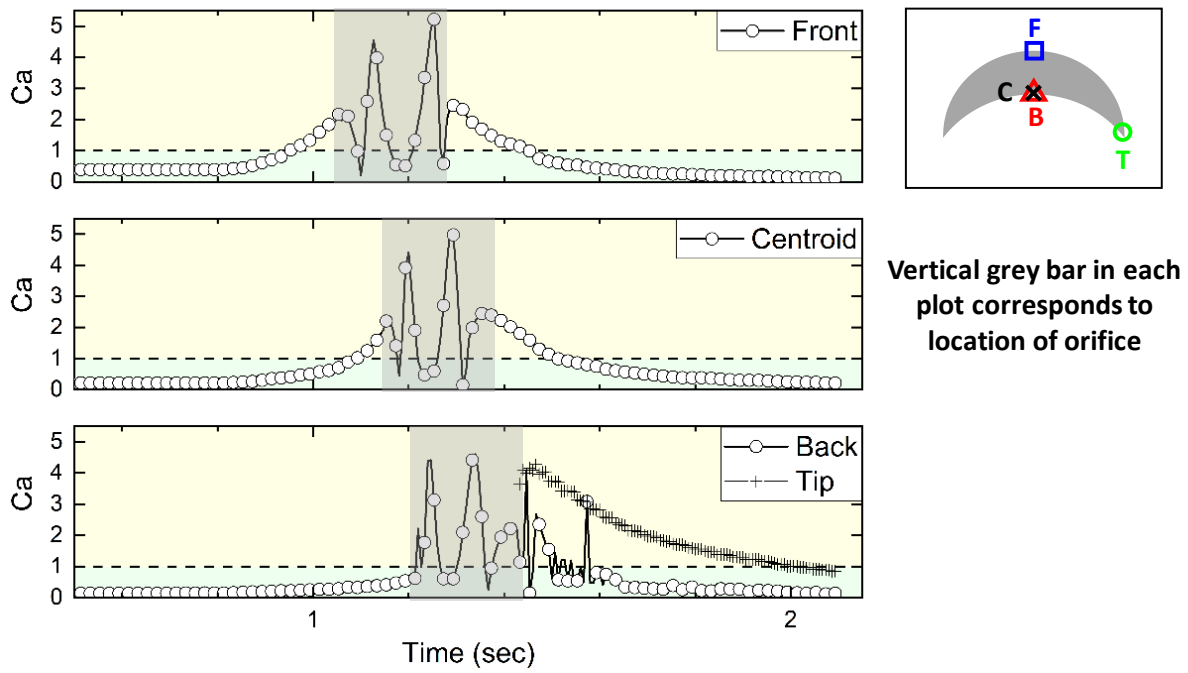


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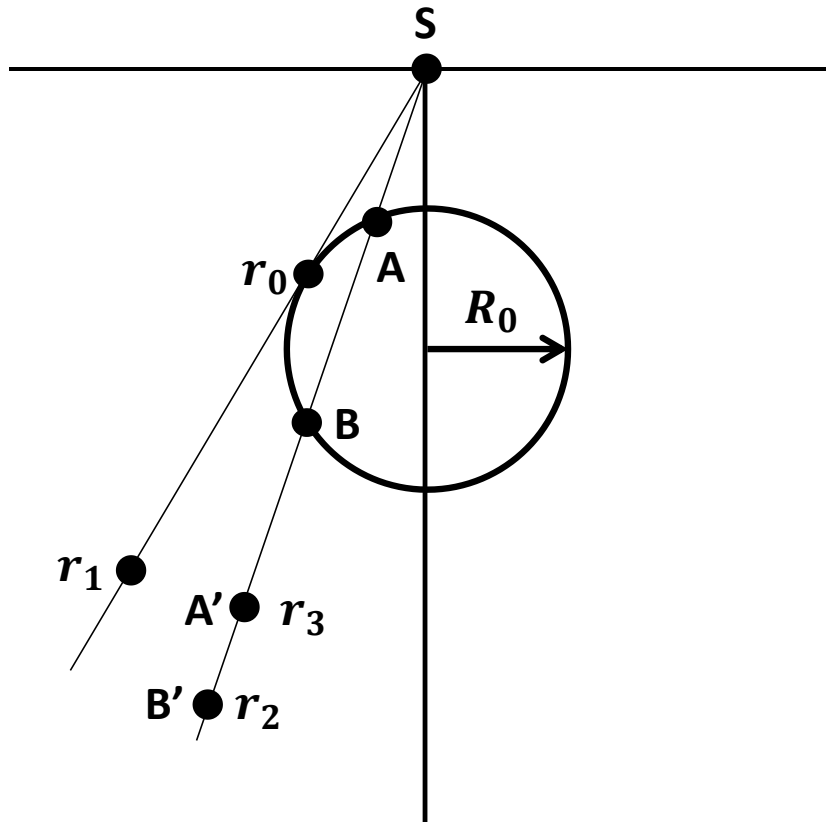


Figure A 1. Movements of elements in radial flow from a point source S . r_2 and r_3 to scale from Equation (5). Movements shown are not to scale.

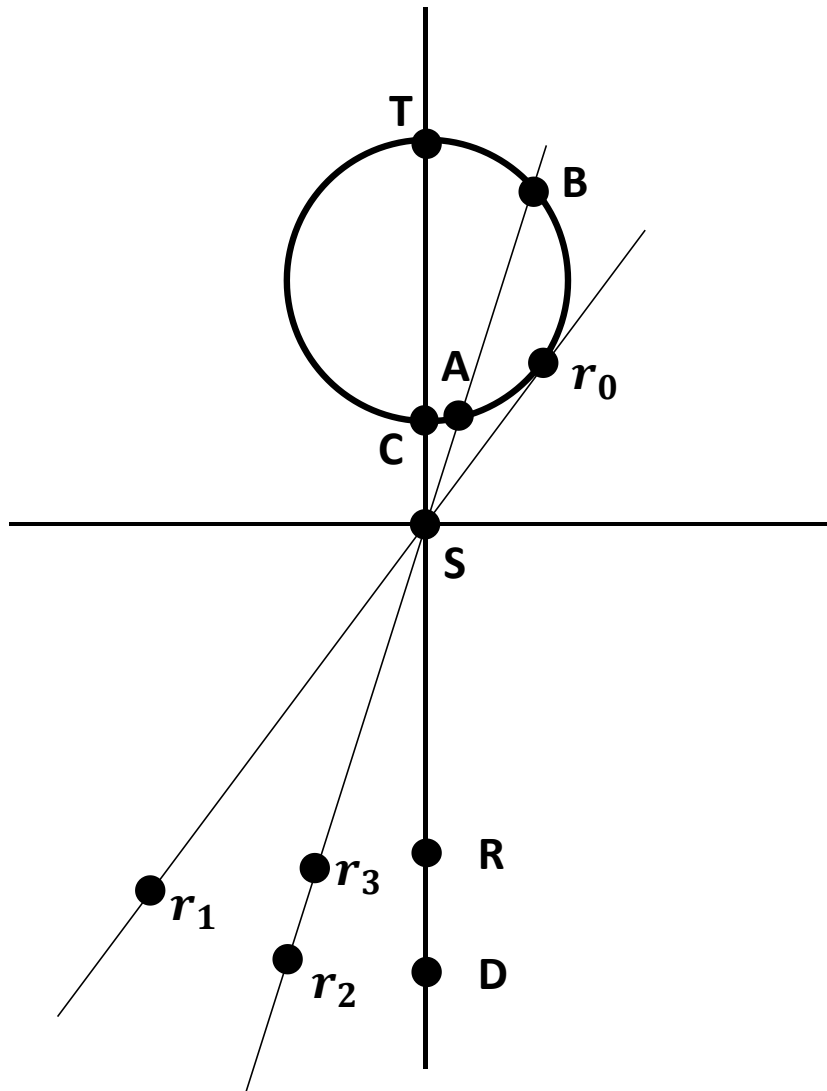


Figure A 2. Bubble trajectory as it passes through an orifice, S .

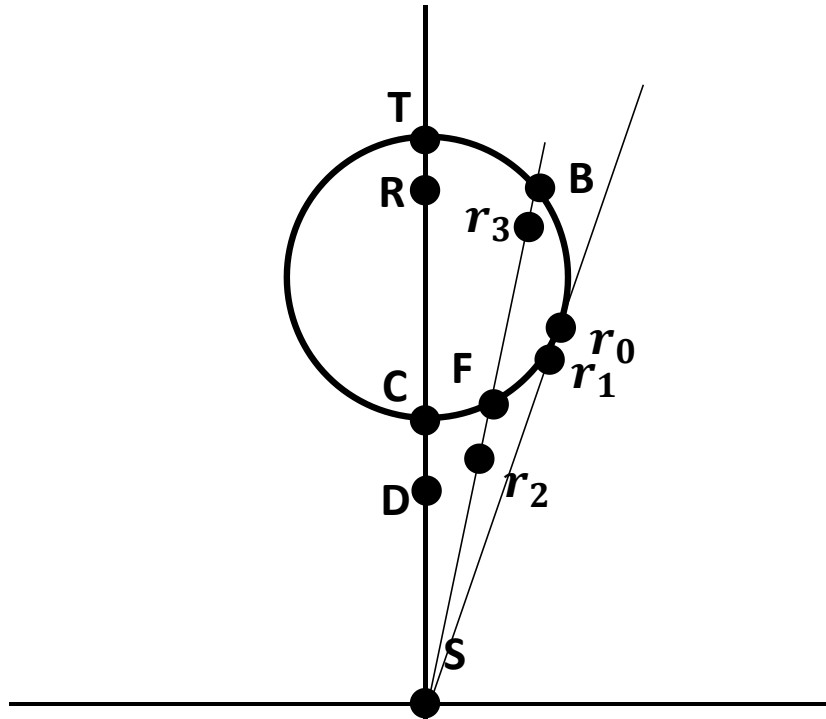


Figure A 3. Bubble approaching an orifice.